Auto-évaluation ex 4 page 115

Sésamath

Maths TS spécialité

énoncé

Soit $a = \dfrac{1+\sqrt{5}}{2}$, $b = \dfrac{1-\sqrt{5}}{2}$ et les matrices :

$$P = \begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix} \text{ et } D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}.$$

1. Vérifier que P est inversible et calculer P^{-1}.
2. Calculer le produit PDP^{-1}.

correction

1

Rappels : inverse d'une matrice 2×2

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- La matrice $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ est inversible si, et seulement si,

$$det(M) = ad - bc \neq 0.$$

correction

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Rappels : inverse d'une matrice 2×2

- La matrice $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ est inversible si, et seulement si,

$$det(M) = ad - bc \neq 0.$$

- Si M est inversible, alors

$$M^{-1} = \frac{1}{det(M)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

correction

1 Or ici,

$$det(P) = a - b = \sqrt{5} \neq 0$$

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Ainsi, P est inversible et

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Ainsi, P est inversible et

$$P^{-1} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -b \\ -1 & a \end{pmatrix}$$

correction

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correction

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$$P = \begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix}$$

correction

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = D$$

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$$P = \begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} & \\ & \end{pmatrix}$$

correction

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = D$$

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$$P = \begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} a \times a + b \times 0 & \\ & \end{pmatrix}$$

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$$P = \begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} a^2 & b^2 \\ a & b \end{pmatrix} = PD$$

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$$PDP^{-1} = \frac{1}{\sqrt{5}} \begin{pmatrix} (a-b)(a+b) & -ab(a-b) \\ a-b & 0 \end{pmatrix}$$

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