Exercice 30 page 371

Sésamath

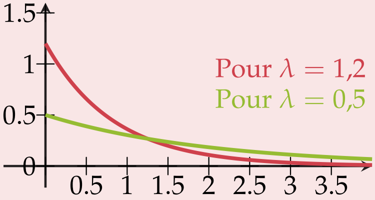
Maths TS obligatoire

Une variable aléatoire X suit la loi exponentielle de paramètre 0,3.
Calculer :

1. $P(X \in [0\,;\,2])$
2. $P(X \in [1\,;\,+\infty[)$
3. $P(5 < X < 10)$
4. $P(X \in [5\,;\,10])$

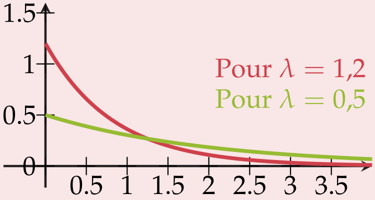
Rappel

Une variable aléatoire X suit la loi exponentielle de paramètre λ où $\lambda > 0$ si elle admet pour densité la fonction f définie sur $[0\,;\,+\infty[$ par $f(x) = \lambda e^{-\lambda x}$.



Rappel

Une variable aléatoire X suit la loi exponentielle de paramètre λ où $\lambda > 0$ si elle admet pour densité la fonction f définie sur $[0\,;\,+\infty[$ par $f(x) = \lambda \mathrm{e}^{-\lambda x}$.



Rappel

Soit X une variable aléatoire suivant la loi exponentielle de paramètre λ et a, c et d trois réels positifs. On a alors :

- $P(c \leqslant X \leqslant d) = \mathrm{e}^{-\lambda c} - \mathrm{e}^{-\lambda d}$
- $P(X \leqslant a) = 1 - \mathrm{e}^{-\lambda a}$
- $P(X \geqslant a) = \mathrm{e}^{-\lambda a}$

1. $P(X \in [0\,;\,2]) = P(0 \leqslant X \leqslant 2)$

correction

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