S'entraîner 53 page 122

Sésamath

Maths 1S

énoncé

Déterminer si les suites (u_n) ci-dessous sont géométriques. Si oui, donner la raison.

1. $\begin{cases} u_0 = -1 \\ u_{n+1} = u_n - \dfrac{1}{4}u_n \end{cases}$

2. $\begin{cases} u_0 = 2 \\ u_{n+1} = 3 + 2u_n \end{cases}$

3. $\begin{cases} u_0 = \dfrac{1}{3} \\ u_{n+1} = \dfrac{1}{2u_n} \end{cases}$

correction

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$u_0 = -1$; $u_1 = u_0 - \dfrac{1}{4}u_0 = -1 - \dfrac{1}{4} \times (-1) = -\dfrac{3}{4}$.

$u_2 = u_1 - \dfrac{1}{4}u_1 = -\dfrac{3}{4} - \dfrac{1}{4} \times \left(-\dfrac{3}{4}\right) = -\dfrac{3}{4} + \dfrac{3}{16} = \dfrac{-9}{16}$

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On observe que : $u_1 = -1 \times \dfrac{3}{4} = u_0 \times \dfrac{3}{4}$ et $u_2 = -\dfrac{3}{4} \times \dfrac{3}{4} = u_1 \times \dfrac{3}{4}$

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Il semble que la suite (u_n) est une suite géométrique de raison $\dfrac{3}{4}$.

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Pour tout entier naturel n, $u_{n+1} = u_n - \dfrac{1}{4}u_n$ donc

$u_{n+1} = u_n \times \left(1 - \dfrac{1}{4}\right) = \dfrac{3}{4}u_n$.

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$\dfrac{u_1}{u_0} = \dfrac{7}{2}$ et $\dfrac{u_2}{u_1} = \dfrac{17}{7}$
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$$\frac{u_1}{u_0} = \frac{\frac{3}{2}}{\frac{1}{3}} = \frac{3}{2} \times 3 = \frac{9}{2} \text{ et } \frac{u_2}{u_1} = \frac{\frac{1}{3}}{\frac{3}{2}} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}.$$

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